t-test

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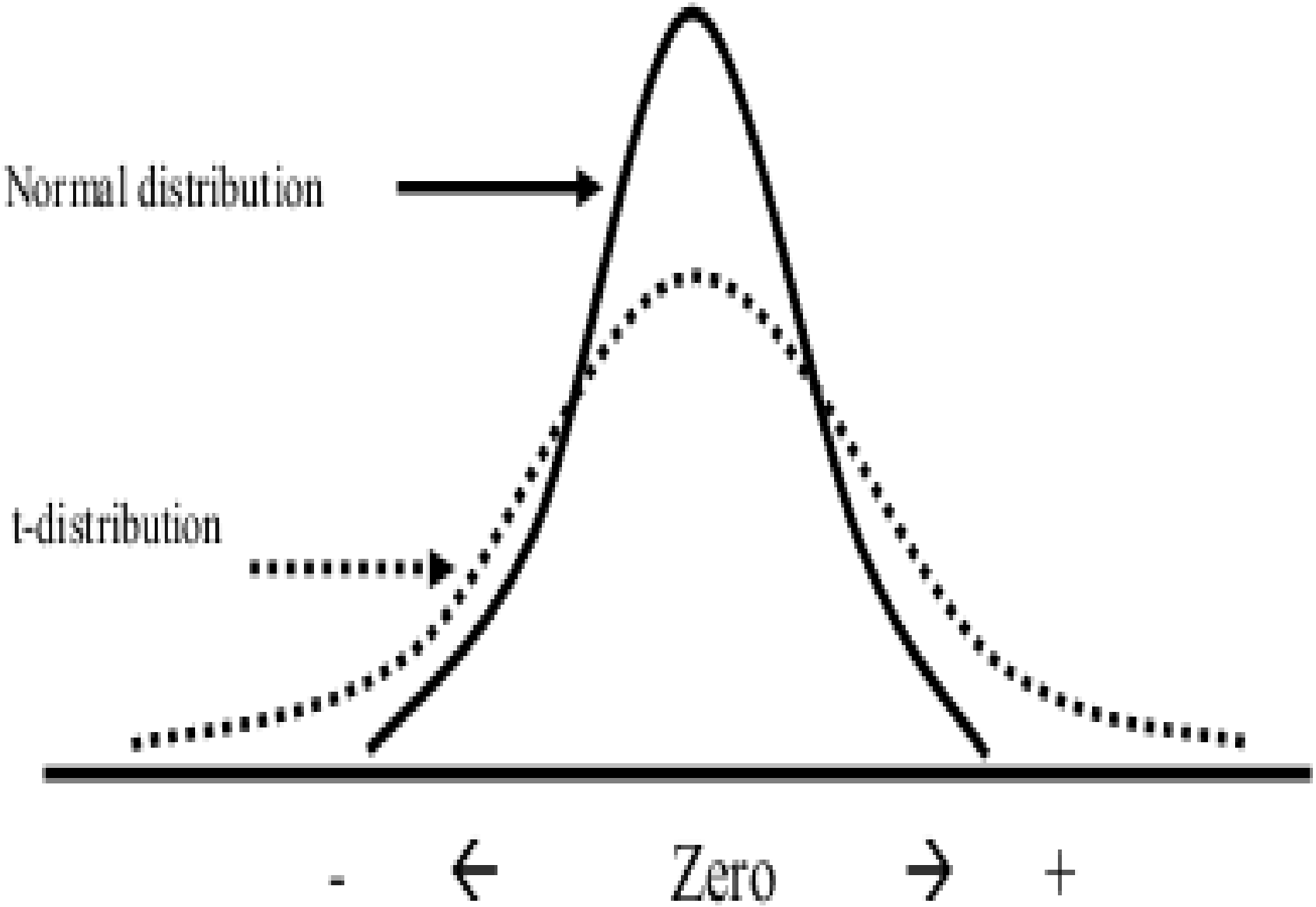
- It is one of the commonly used tests for testing hypothesis for testing the significance of difference for the quantitative data. It depends on a distribution called the t-distribution with (n-1) degree of freedom. This distribution was introduced by William S Gosset, who used the pen-name “student” and is often called students’ t-test in his honor (or t-test), distribution which is like the normal distribution of a symmetrical bell-shaped distribution with a mean of zero but it is of lower peak, higher tails, and more spread out (more probability in the tails and less in the center), having two tails.


Normal distribution
t-distribution
-
Zero
+

- The exact shape of the t-distribution depends on the degree of freedom (d.f.= n-1), and on the sample SD, the fewer the degrees of freedom, the more the t-distribution is spread out. t-test use is restricted to the small sample size (less than 30).
- t-test represent the measurement of the significancy of difference between two means;
- $t = \frac{Difference\ between\ two\ means}{Standard\ error\ of\ difference}$

- **Applications of t-test;**

1. For calculation of population mean.
2. For calculation of significance of difference between sample mean and population mean.
3. For calculation of significance of difference between two independent means.
4. For calculation of significance of difference between two dependent means (paired observations).

1- Calculation of population mean:

- In general, a confidence interval "C.I." (Population mean; μ) is calculated using t distribution through appropriate significance level (α=0.05 for 95% C.I., α=0.01 for 99% C.I.) with (n-1) degrees of freedom. This is applied for small sample size (n < 30) because for large degrees of freedom, the t distribution is almost the same as the standard normal distribution.

- **e.g.** The followings are the numbers of hours of relief obtained by 6 patients after receiving a new drug;
- 2.2, 2.4, 4.9, 2.5, 3.7, & 4.3 hours
- Mean = 3.3 hours SD= ±1.3 hours n= 6
- Calculate population mean? (Using α=0.05).
- Tabulated t for α=0.05, for d.f. (n-1) is;
- α=0.05
- t = 2.571
- df.=n-1=6-1=5
- μ = mean ± x SE
- $\mu = 3.3 \pm 2.57 \times (1.13/\sqrt{6})$
- $\mu = 3.3 \pm 2.57 \times 0.46$
- $\mu = 3.3 \pm 1.2$
- Lower limit = 3.3-1.2 → 2.1 hours
- Upper limit = 3.3+1.2 → 4.5 hours
- μ → (2.1 --- 4.5) hours

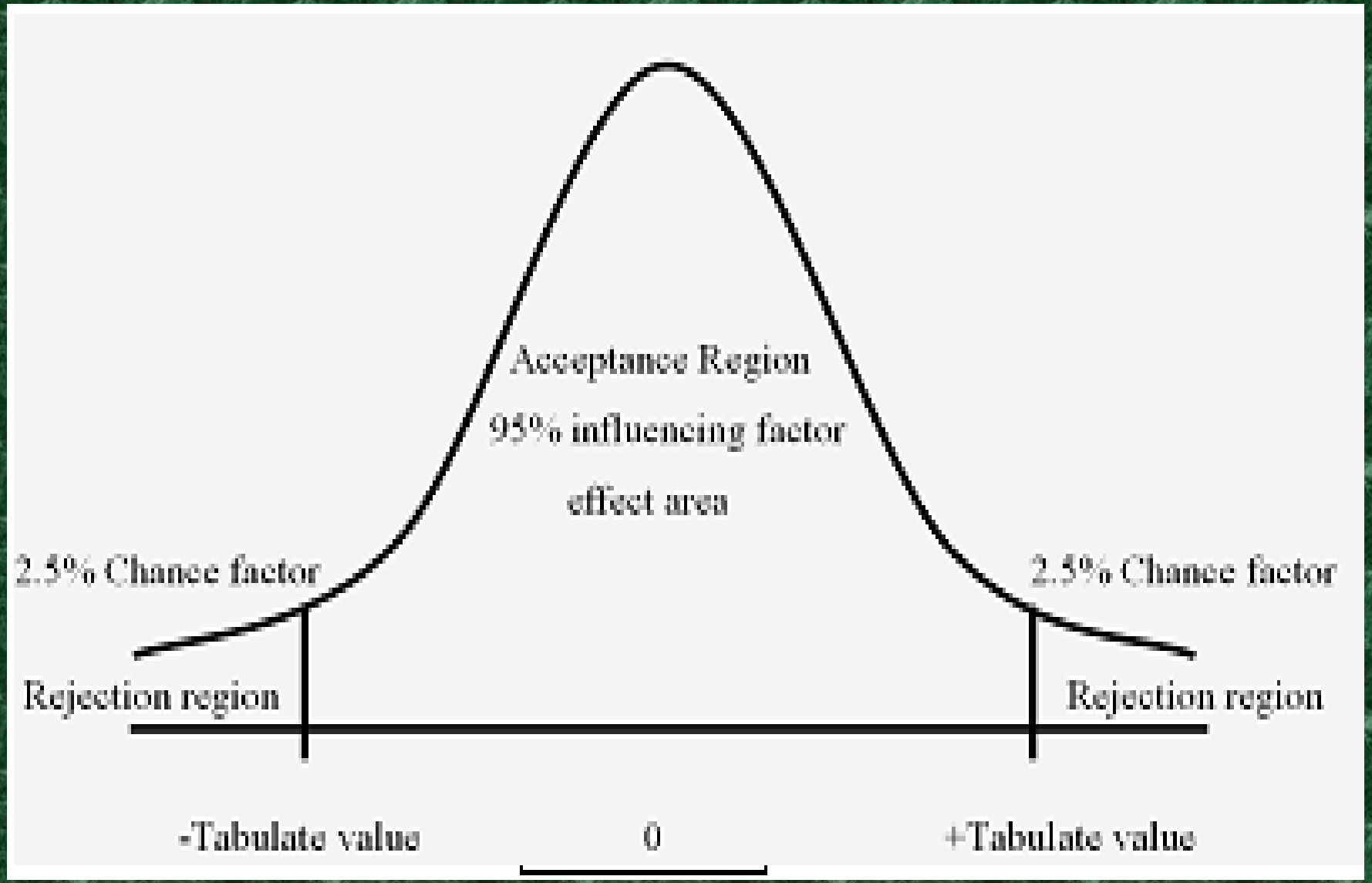
- **2-Difference between sample mean and population mean:**
- The t-test is used to test the significance of the difference of sample mean from a population mean or a standard mean or a standard value.
- **e.g.** The followings are the heights in cm of 24 two-years-old boys with sickle cell disease.

84.4	89.9	89.0	81.9	87.0	78.5	84.1	86.3
80.6	80.0	81.3	86.8	83.4	89.8	85.4	80.6
85.0	82.5	80.7	84.3	85.4	85.0	85.5	81.9

- Height standard for United Kingdom (U.K.) give a reference height for two-years-old boys of 86.5 cm (representμ).
- Does the above sample suggest that the two-years-old boys with sickle cell (SC) disease differ in height from the standards? (use $\alpha=0.05$).

- **Data:** Data represent the heights of 24 two-years-old boys with SC disease, with; Mean = 84.1 cm
- SD= ±3.11 cm → SE=SD/√n = 3.11/√24=3.11/4.9= ±0.63 cm
- **Assumption:** We assume that the sample of 24 two-years-old boys with SC disease was selected randomly from a normally distributed population of boys with SC disease.
- **H_o:** There is no significant difference between mean height of boys with SC disease from the normal standard height ($m1=\mu$; $m1-\mu=0$). OR There is no significant influence (effect) of SC disease on the height of children.

- **H_A:** There is significant difference between mean height of boys with SC disease from the normal standard height (m1 μ; m1- μ 0). OR There is significant influence (effect) of SC disease on the height of children.
- **Level of significance;** (α = 0.05); 5% Chance factor effect area 95% Influencing factor effect area (SC disease) d.f.=n-1; tabulated t for d.f {(n-l)=24-1=23} equal to 2.069.


Acceptance Region
95% influencing factor
effect area
2.5% Chance factor
2.5% Chance factor
Rejection region
Rejection region
-Tabulate value
0
+Tabulate value

-2.069 +2.069

- **Apply the proper test of significance**

$$t = \frac{\text{Difference between two means}}{\text{Standard error of difference}}$$

$$t = \frac{m - \mu}{\text{standard error of sample}}$$

$$t = \frac{m - \mu}{\text{Standard error of sample}}$$

$$= \frac{84.1 - 86.5}{0.63} = -3.81 \quad \text{(calculated t value)}$$

$3.81 > 2.069$

Since Calculated t > Tabulated t

So P<0.05

Then reject Ho and accept HA ....

- There is significant difference between mean height of boys with SC disease from the normal standard height.
- There is significant influence (effect) of SC disease on the height of Children.
- Significantly SC disease lowering height of children.
- ⟶ There is stunting of height due to the effect of SC disease.

α=0.05	α=0.02	α=0.01	α=0.001	
t = 2.069	t = 2.50	t = 2.81	t = 3.77	
df.=23	df.=23	df.=23	df.=23	
P value < 0.05	<0.02	<0.01	<0.001	Highly significant effect

- **3-Difference between two independent means:**
- The difference between the means of two independent samples is normally distributed. The same procedure for calculation is followed except that;

1. Assumption; we assume that we have two independent samples randomly chosen each one from a normally distributed population with equal variances of populations.
2. Equation… as the SE of difference is calculated as; Standard deviation of population (pooled SD) ⟶ SP

$$SP = \sqrt{\frac{S_1^2 (n1 - 1) + S_2^2 (n2 - 1)}{n_1 + n_2 - 2}}$$

S_1^2 → variance of group 1

S_2^2 → variance of group 2

$$\text{SE of difference} = SP \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

3- The **d.f.** $= n_1 + n_2 - 2$ or $(n_1 - 1) + (n_2 - 1)$

e.g. A study of birth weight of infants born to 15 non-smoker and 14 heavy smoker mothers (during pregnancy).

Birth weight (Kg) of infants born to non-smoker mothers (n=15)	Birth weight (Kg) of infants born to heavy smoker mothers (n-14)
3.99	3.18
3.79	2.84
3.60	2.90
3.73	3.27
3.21	3.85
3.60	3.52
4.08	3.23
3.61	2.76
3.83	3.60
3.31	3.75
4.13	3.59
3.26	3.63
3.54	2.38
3.51	2.34
2.71	
$Mean_1 = 3.5933$ Kg	$Mean_2 = 3.2029$ Kg
$SD_1 = \pm 0.37$ Kg	$SD_2 = \pm 0.49$ Kg
$n_1 = 15$	$n_2 = 14$

Does these data provide any evidence of effect of smoking during pregnancy on birth weight? (use $\alpha=0.05$).

- **Data**: Data represent the birth weight in kilograms of two independent groups of smoker and non-smoker mothers infants with mean birth weight of non-smoker mother's infants of 3.5933, and of heavy smoker mother's infants of 3.2029 kilograms.
- **Assumption**: We assume that the two independent groups (of infants born to non-smoker mothers and those born to heavy smokers mothers) were randomly drawn each one form a normally distributed population with equal variances of populations.

- **H_O**: There is no significant difference between mean birth weight of infants of non-smoker mothers and birth weight of infants of heavy smoker mothers ($m_1=m_2$; $m_1- m_2=0$). **OR** There is no significant influence (effect) of mother smoking during pregnancy on birth weight of their infants.
- **H_A**: There is significant difference between mean birth weight of infants of non-smoker mothers and birth weight of infants of heavy smoker mothers ($m_1\neq m_2$; $m_1- m_2\neq 0$). **OR** There is significant influence (effect) of mother smoking during pregnancy on birth weight of their infants.

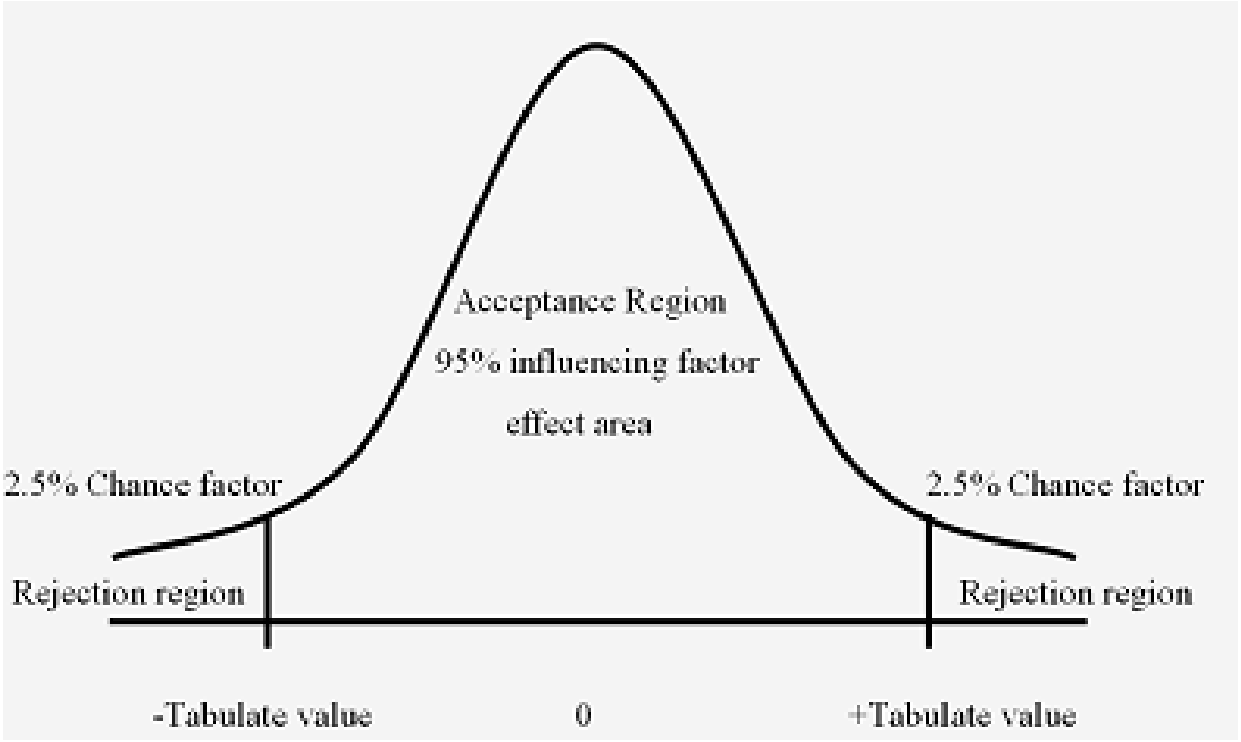
- **Level of significance**; ($\alpha = 0.05$);

  5% Chance factor effect area

  95% Influencing factor effect area (SC disease)

  d.f.= $n_1 + n_2 - 2$ or $(n_1-1) + (n_2-1)$;

  tabulated t for d.f ($n_1 + n_2 - 2$ =15+14-2=27) equal to 2.052



- **Apply the proper test of significance**

$$t = \frac{\text{Difference between two means}}{\text{Standard error of difference}}$$

$$t = \frac{m_1 - m_2}{\text{standard error of difference (Pooled SE)}}$$

$$t = \frac{m_1 - m_2}{\text{Standard error of difference (Pooled SE)}}$$

$$SP = \sqrt{\frac{S_1^2 (n1 - 1) + S_2^2 (n2 - 1)}{n_1 + n_2 - 2}} = \sqrt{\frac{[(0.37^2 \times (15-1) + 0.49^2 \times (14-1)]}{15+14-2}}$$

$$SP = \pm 0.43 \text{ Kg}$$

$$t = \frac{m_1 - m_2}{SP \sqrt{(1/n_1) + (1/n_2)}} = \frac{3.5933 - 3.2029}{0.43 \times \sqrt{(1/15)+(1/14)}} = 2.42 \text{ (calculated t value)}$$

$2.42 > 2.052$

Since Calculated t > Tabulated t

So $P < 0.05$

Then reject Ho and accept HA ....

- There is significant difference between mean birth weight of infants of non-smoker mothers and birth weight of infants of heavy smoker mothers.
- There is significant influence (effect) of mother smoking during pregnancy on birth weight of their infants
- Meaning there is significanctly lowering of birth weight of infants by the effect of smoking of their mothers during pregnancy.

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α=0.05	α=0.02	α=0.01	α=0.001
t = 2.05	t = 2.47	t = 2.77	t = 3.69
df.=27	df.=27	df.=27	df.=27

P value < 0.05 <0.02 significant effect

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Table 9.4. Critical values of the probability distribution of $T = |t_{(k)}|$; the table specifies values of the number t_p such that $Pr(T > t_p) = p$

Degrees of freedom (k)	Probability level, p					
	0.10	0.05	0.02	0.01	0.002	0.001
1	6.314	12.706	31.82	63.66	318.3	636.6
2	2.920	4.303	6.695	9.925	22.33	31.60
3	2.353	3.182	4.541	5.841	10.21	12.92
4	2.132	2.776	3.747	4.604	7.173	8.610
5	2.015	2.571	3.365	4.032	5.893	6.869
6	1.943	2.447	3.143	3.707	5.208	5.959
7	1.895	2.365	2.998	3.499	4.785	5.408
8	1.860	2.306	2.896	3.355	4.501	5.041
9	1.833	2.262	2.821	3.250	4.297	4.781
10	1.812	2.228	2.764	3.169	4.144	4.587
11	1.796	2.201	2.718	3.106	4.025	4.437
12	1.782	2.179	2.681	3.055	3.930	4.318
13	1.771	2.160	2.650	3.012	3.852	4.221
14	1.761	2.145	2.624	2.977	3.787	4.140
15	1.753	2.131	2.602	2.947	3.733	4.073
16	1.746	2.120	2.583	2.921	3.686	4.015
17	1.740	2.110	2.567	2.898	3.646	3.965
18	1.734	2.101	2.552	2.878	3.610	3.922
19	1.729	2.093	2.539	2.861	3.579	3.883
20	1.725	2.086	2.528	2.845	3.552	3.850
21	1.721	2.080	2.518	2.831	3.527	3.819
22	1.717	2.074	2.508	2.819	3.505	3.792
23	1.714	2.069	2.500	2.807	3.485	3.767
24	1.711	2.064	2.492	2.797	3.467	3.745
25	1.708	2.060	2.485	2.787	3.450	3.725
26	1.706	2.056	2.479	2.779	3.435	3.707
27	1.703	2.052	2.473	2.771	3.421	3.690
28	1.701	2.048	2.467	2.763	3.408	3.674
29	1.699	2.045	2.462	2.756	3.396	3.659
30	1.697	2.042	2.457	2.750	3.385	3.646
40	1.684	2.021	2.423	2.704	3.307	3.551
60	1.671	2.000	2.390	2.660	3.232	3.460
120	1.658	1.980	2.358	2.617	3.160	3.373
∞ (normal)	1.645	1.960	2.326	2.576	3.090	3.291

4-Difference between two dependent sample means:

The use of t-test here for the difference between pairs of variables measured on each individual, such as the results of blood pressure of each individual before taking hypotensive agent and after its taken, if the drug have no effect so there is no difference in the values of blood pressure before and after its taken (as there is small but not significant difference), but if there is effect of drug so there is difference between the two measurements, it means there is significant difference (before and after). So the same sample (one group) under two occasions is considered here, e.g. before and after, etc…

The use of test is restricted for samples with matched pairs of less than 30. This technique is applied in order to eliminate as much as possible a maximum number of sources of variation by making the pairs similar (identical) to each other with respect to as many variables as possible (for the same persons two measurements one before and one after as the difference is mostly due to the effect of the factor itself)

e.g. The effect of a certain sleeping drug is to be tested by taking 10 patients, we gave them at first night a placebo drug and we measure the sleeping hours, next night we gave them the sleeping drug and we measure the sleeping hours, the following results were obtained from this placebo-controlled clinical trial to test the effectiveness of a sleeping drug;

patient	Hours of sleep	
	drug	placebo
1	6.1	5.2
2	7.0	7.9
3	8.2	3.9
4	7.6	4.7
5	6.5	5.3
6	8.4	5.4
7	6.9	4.2
8	6.7	6.1
9	7.4	3.8
10	5.8	6.3

What can you conclude from that data? (Use α 0.05)

The answer;

patient	.d (drug-placebo)	$.d^2$
1	6.1-5.2=+0.9	0.81
2	-0.9	0.81
3	+4.3	18.49
4	+2.9	8.41
5	+1.2	1.44
6	+3.0	9.00
7	+2.7	7.29
8	+0.6	0.36
9	+3.6	12.96
10	-0.5	0.25
	$\sum d=17.8$	$\sum d^2=59.82$

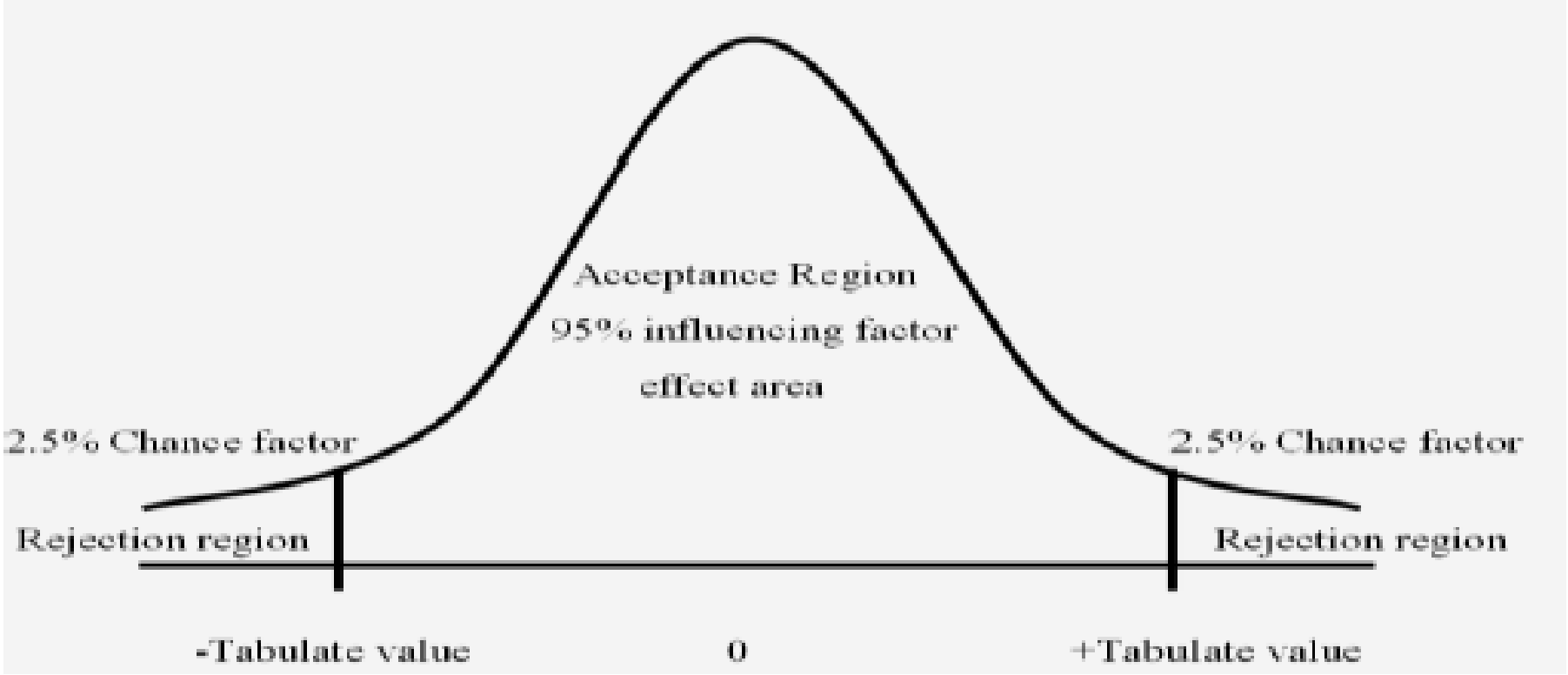
- **Data**: Data represent 10 patients with sleeping hours for each one before and after taking a sleeping agent, with mean duration of sleeping of 7.06 hours after taking the sleeping agent, and 5.28 hours after taking a placebo.
- **Assumption**: We assume that the sample was selected randomly from a normally distributed population. **OR** we assume that the observed difference constitutes a simple random sample from a normally distributed population of differences.

- **HO**: There is no significant difference in the mean sleeping hours after taking the drug from that of the placebo (m1=m2; m1-m2=0). **OR** There is no significant effect of the drug as a sleeping agent.
- **HA**: There is significant difference in the mean sleeping hours after taking the drug from that of the placebo (m1 ≠ m2; m1-m2 ≠ 0). **OR** There is significant effect of the drug as a sleeping agent
- **Level of significance**; ($\alpha = 0.05$);

  5% Chance factor effect area

  95% Influencing factor effect area (sleeping agent) d.f. =n-1;

tabulated t for d.f {(n-l)=l0-1=9} equal to 2.2622



-2.2622

+2.2622

- **Apply the proper test of significance**

$$\text{sd of difference} = \sqrt{\left(\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}\right)} = \sqrt{\{(59.82 - 17.8^2/10)/9\}} = \pm 1.77$$

$$t = \frac{m1 - m2}{SD_{\text{of difference}}/\sqrt{n}} = \frac{7.06 - 5.28}{1.77/\sqrt{10}} = \frac{1.78}{0.56} = 3.1786 \text{ (calculated t)}$$

$3.1786 > 2.2622$

Since Calculated t > Tabulated t So $P<0.05$

Then reject Ho and accept HA ....

- There is significant difference in the mean sleeping hours after taking the drug from that of the placebo
- There is significant effect of the drug as a sleeping agent
- Significantly this agent is increasing the sleeping hours after its taken.

- **Reasons for pairing;**

It frequently true that true differences do not exist between two populations with respect to the variable of interest, but the presence of other (extraneous) sources of variations may cause rejection of the hypothesis of no difference. On the other hand true differences may also be masked by the presence of extraneous factors.

Therefore the **objective** in paired comparison tests;

- is to **eliminate** a maximum number of sources of extraneous variation by making the pairs similar with respect to as many variables as possible
- **Fewer** subjects are used with repeated measurements on each one, this is of value specially if subjects are scare or expensive to recruit.

Disadvantages of paired comparisons;

- Considerable **time** and **expenses** are needed in our trying to match individuals on one or more relevant variables.
- **Loss of degrees of freedom**. If we do not use paired observations, we have 2n-2 or n1+n2-2 as compared to n-1 when we use the paired comparison procedure.
- **Carry-over effect**; especially when one or more treatments are being evaluated with possibility of residual effect from previous treatment. This problem can frequently be solved by allowing a sufficient length of time between treatments.
- **Position effect**; A subject response to a treatment experienced last in a sequence may be different from the response that would have occurred if the treatment had been first in the sequence. This problem can be solved by randomize the sequence of treatments independently for each subject.

END