PHYSICS OF NUCLEAR MEDICINE

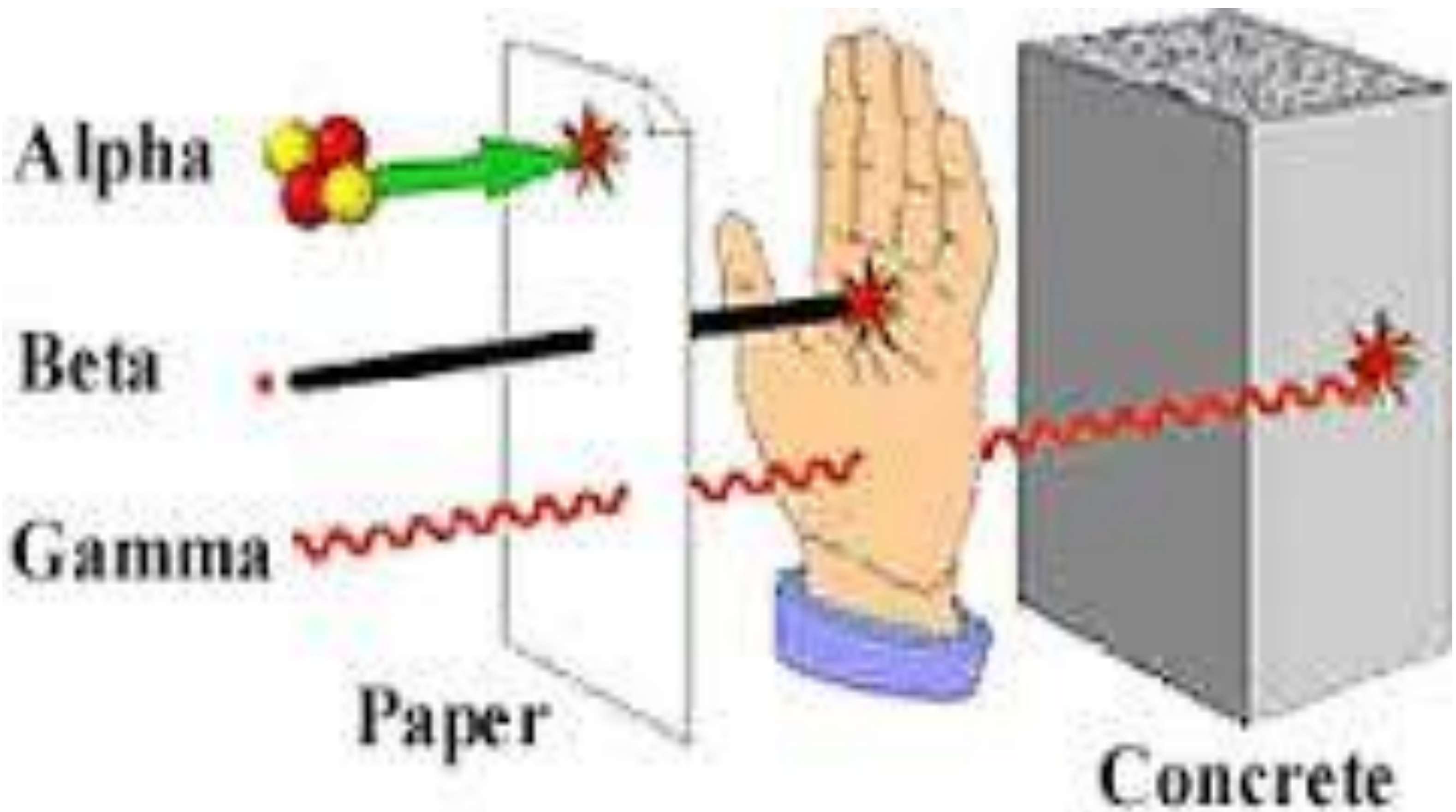
Physics of nuclear medicine
By the end of this section, you will be able to:

1- Define the half-life.
2- Give the essential characteristics of alpha, beta, and gamma rays.
3- What is the mass of 1ci of 227Th? If the half-life is 1.90 years.

Radioactivity

A certain natural elements, heavy have unstable that disintegrate to emit various rays. Alpha(α), Beta(β), and Gamma(γ)rays.

Alpha(α)	Beta(β)	Gamma(γ)
1-Positive charge	Negative charge	Without charge
2-Affected by magnetic& electric field	Affected by magnetic& electric field	Doesn’t affected
3-Stop in a few centimeter of air (low penetrating power	It is stopped in a few meters of air and a few millimeters of a tissue(the penetrating power is more than α and less than γ	High energy photon(high penetrating power).
4-Is Helium atom($_2He^4$)	High speed electron	It is photon
5-Has a fixed energy for a given source	Has spread of energy up to max	Has a fixed energy for a given source


Alpha
Beta
Gamma
Paper
Concrete

Isotpes

Nuclei of a given element with different numbers of neutrons.

There are two types:

1-Stable isotopes if they are not radioactive. Ex:(^{12}C,^{13}C)

2-Radioisotopes if they are radioactive. Ex: (^{11}C,^{14}C,^{15}C)

Radio-nuclides:

Is used when several radioactive elements are involved.(Radioisotopes are used when referring to single element).

Neutrino:

A mass less, charge less, particle, Takes up the difference in energy between the actual beta energy and the maximum beta energy.

Alpha (α):

Is helium atom($^{4}_{2}He$) with mass number (A) = 4 and atomic number (Z) = 2. The result of alpha emission is a daughter whose atomic number is two less than of the parent, and whose atomic mass number is four less than that of the parent. In the case of ^{210}Po for example, the reaction is

$$^{210}_{84}Po \longrightarrow {}^{4}_{2}He + {}^{206}_{82}Pb$$

Or in general

$$^{A}_{Z}X \longrightarrow {}^{4}_{2}He + {}^{A-4}_{Z-2}Y$$

Beta emission:

$$^{32}_{15}P \longrightarrow ^{32}_{16}S + ^{0}_{-1}e$$

Or positron

$$^{22}_{11}Na \longrightarrow ^{22}_{10}Na + ^{0}_{1}e$$

Activity of Radioactive materials

-half-life

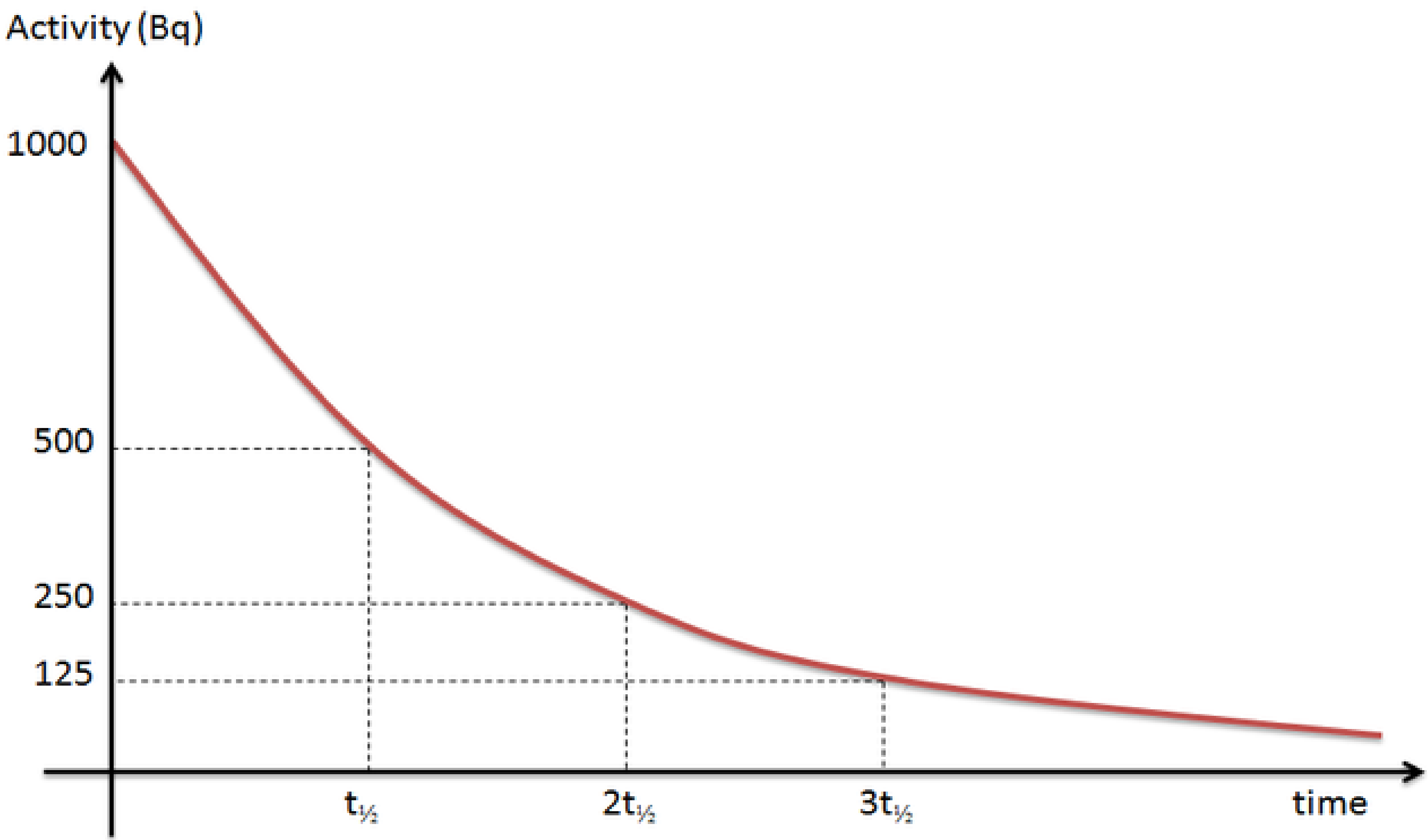
-mean life

-decay constant

-background

Half life ($T_{1/2}$):

The time needed for half of the radioactive nuclei to decay.



$$A = A_0 \, e^{-\lambda t} \quad \text{………. (1)}$$

Where:

A : activity in disintegration per second after time(t)

A_o: initial activity

λ : decay constant(sec^{-1},$hour^{-1}$,$year^{-1}$)

t : time since activity (sec, hour, year)

$$T_{1/2} = 0.693 / \lambda \quad \ldots\ldots\ldots\ldots \quad (3)$$

$A = \lambda N = (0.693/ T_{1/2}$(mass/atomic weight)x Avogadro number

1 year = 3.15 x 10^7sec

$T_{1/2}$ = should be in second

The average or mean time T = 1/ λ

1/ λ from the equation (3) = 1.44 $T_{1/2}$

So T = 1.44 $T_{1/2}$

T mean life time (tau) is the average lifetime of a radioactive particle before decay.

Example 1.

a. If you have 1g of pure potassium 40 (^{40}K) that is experimentally determined to emit about 10^5 beta rays per second. What is the decay constant λ?

Solution:

A = λN = λ (mass/atomic weight)x Avogadro number)

$10^5 = \lambda \times 1/40 \times 6.02 \times 10^{23}$

So $\lambda = 6.7 \times 10^{-18}\ s^{-1}$

b. Estimate the half-life of ^{40}K from .

$T_{1/2} = 0.693/\lambda = 10^{17}$

$T_{1/2} = 10^{17}/3.15 \times 10^7 = 3 \times 10^9$ years

Back ground counts:

Is the counts without the radioactive source and this is due cosmic rays, natural radioactivity ……etc

Units of activity:

The unit of activity of radioactive is Ci (Curie)

1 Ci = 3.7 x 10^{10} dis/s of Bq (Becquerel)

(micro)μ Ci = 10^{-6}Ci

(nano)η Ci = 10^{-9}Ci

(pico)ρ Ci = 10^{-12} Ci

Questions:

1- A solution counting a radioactive isotope which emits β-particles with half-life 12.26 days' surroundings a Geiger counter which records 480 counts/minute. What counting rate will be obtained 49.04 days later?

Solution

A_o = 480 counts/min

A = ?

t = 49.04 days

$T_{1/2}$ = 12.26 days

$\lambda = 0.693/\ T_{1/2} = 0.693/12.26$ days

$A = A_0\ e^{-\lambda t}$

A =480 counts/min x $e^{-(0.693/12.26 \text{ days})\ 49.04 \text{ days}}$

A = 480 counts/min x e -4(0.693)

A = 480 counts/min x $1/\ 2^4$

A = 480/16 = 30 counts/min

2- Radium 226 has a half life of 1620 years. What is the mass of a sample which undergoes 20000 disintegrations per second?

Solution:

$T_{1/2}$ = 1620 years = 1620 x 3.15 x 10^7 s

$\lambda = 0.693/ T_{1/2} = 0.693/(1620 \times 3.15 \times 10^7$ s)

A = 2x 10^4 dis/s

A = N λ

2x 10^4 dis/s = (m/226) x 6.02 x 10^{23} x 0.693/(1620 x 3.15 x 10^7 s)

m = 55x 10^{-6}g

3- What is the mass of 1ci of ^{227}Th? If the half-life is 1.90 years.

Solution:

$T_{1/2}$ = 1.90 years = 1.90 x 3.15 x 10^7 s

λ = 0.693/ $T_{1/2}$ = 0.693/(1.90 x 3.15 x 10^7 s)

A = 1 ci = 3.7 x 10^{10} dis/s

A = N λ

3.7 x 10^{10} dis/s = (m/227)x6.02x10^{23} x 0.693/(1.90 x 3.15 x 10^7 s)

m = 1.21x 10^{-3}g

4- Iodine-131 is used to destroy thyroid tissue in the treatment of an overactive thyroid. The half – life of ^{131}I is 8 days. If a hospital receives a shipment of 200g of ^{131}I, how much ^{131}I would remain after 32 days?4

Solution:

$\lambda = 0.693/ T_{1/2} = 0.693/8\text{days}$

$t = 32$ days

$A = A_0\, e^{-\lambda t}$

$m = m_0\, e^{-\lambda t}$

$m = 200\text{g} \times e^{-(0.693/8\text{days}) \times 32\text{days}}$

$m = 200\text{g} \times e^{-4(0.693)} = 200\text{g} \times 1/16 = 12.5\text{g}$

5- If 10mg of iodine-131 is given to a patient, how much is left after 24 days? The half – life of ^{131}I is 8 days.

Solution:

$t = 24days$

$\lambda = 0.693/\ T_{1/2} = 0.693/8days$

$m = m_0\ e^{-\lambda t}$

$m = 10x\ 10^{-3}g\ x\ e^{-(0.693/8days)x24days}$

$m = 10^{-2}g\ x\ e^{\ -3(0.693)} = 10^{-2}g\ x\ 1/8 = 1.25$
$x10^{-3}g = 1.25mg$

6- Technetium -99m (^{99m}Tc) is used for brain scans, if a laboratory receives a shipment of 200gm of this isotope and after 24 hours only 12.5 g of this isotope remain, what is the half-life of ^{99m}Tc.

$m_o = 200g$

$m = 12.5g$

$t = 24$ hr

$m = m_0\ e^{-\lambda t}$

$m_o/m = e^{\lambda t}$

$200/12.5 = e^{24\lambda}$

$16 = e^{24\lambda}$

$\text{Log } 16 = 24\ \lambda\ \text{Log } 2.7$

$\lambda = 0.116\ hr^{-1}$

$T_{1/2} = 0.693/0.116 = 6$ hr

7- Mercury-197 is used for kidney scans and has a half-life of 3 days. If the amount of mercury -197 needed for a study is 1.0 g and the time allowed for shipment is 15 days, how much mercury -197 will need to be ordered.

$m = 1g$

$m_o = ?$

$T_{1/2} = 3$ days

$\lambda = 0.693/3days$

$m_o/m = e^{\lambda t}$

$m_o/1 = e^{(0.693/3days)15days}$

$m_o = 32g$

8- The half –life of strontium – 90 is 25 years, how much half-life will it take for 10g of it to be reduced to 1.25g.

<u>Answer</u>: 3 $T_{1/2}$

9-The half-life of ^{99m}Tc is 6 hours, after how much time will 1/16 of the radioisotope remain.

Answer: 24 hours

10- Radioactive ^{24}Na, which has a half life of 15 h, is sent from laboratory to a hospital . What should be its activity when it leaves laboratory if the activity is to be 10mCi (milli curies) when it used in the hospital 3 h later.

Answer: 11.5mCi